Published by Institute of Physics Publishing for SISSA/ISAS
Received: *April 28, 2003*
Revised: *November 10, 2003*
Accepted: *December 8, 2003*

D-brane anti-brane annihilation in an expanding universe

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Abstract: The time-varying density of D-branes and anti-D-branes in an expanding universe is calculated. The D-brane anti-brane annihilation rate is shown to be too small to compete with the expansion rate of a FRW type universe and the branes over-close the universe. This brane problem is analogous to the old monopole problem. Interestingly however, it is shown that small dimension D-branes annihilate more slowly than high dimension branes. Hence, an initially brany universe may be filled with only low dimension branes at late times. When combined with an appropriate late inflationary theory this leads to an attractive dynamical way to create a realistic braneworld scenario.

Keywords: D-branes, Tachyon Condensation, Cosmology of Theories beyond the SM, Physics of the Early Universe.

Contents

1. Introduction

In a previous article we showed how D-branes may have been produced in a tachyonic phase transition at an energy near the string scale [1]. While providing a well-defined non-perturbative method of generating branes, this mechanism suffers from the typical problem common to all topological defect theories — too many defects are produced.

The mass of a D-brane is proportional to its volume and is inversely proportional to the closed string coupling g_s. Thus non-compact branes at weak coupling are extremely heavy. Compact branes which wrap compact cycles of a spacetime M are less heavy, but are still much heavier than fundamental string states like electrons and heavy gauge bosons, etc. Thus, a gas of branes, whether compact or non-compact, is likely to cause the energy density of the universe to exceed the critical energy density and turn an expanding universe into a contracting one [2].

For reasonable values of $g_s$branes will distort any cosmological spacetime and dilution their via brane anti-brane annihilation, inflation, or something similar is important.[1] In

[1]If $g_s \to 0$, branes do not pose many cosmological problems because they then decouple from gravity in bulk. This is albeit that the mass $m_{Dp} \sim V_p/g_s$ diverges. Gravity couples to D-branes through closed string emission and absorption. The "vertices" (in field theory parlance) for such interactions are generated by closed string vertex operators, $V(\mathbf{k})$, which are proportional to g_s. Thus as $g_s \to 0$, the vertex operators "tend to" zero, closed string emission/absorption is suppressed, and gravity decouples from the branes. This is similar to the fact that the vertex for electron-photon interactions is proportional to the fine-structure constant α; and if $\alpha \to 0$ then electrons and photons no longer interact. Thus interestingly, heavy branes

this paper we try to determine how brane anti-brane annihilation affects brane densities. Such a calculation is useful to: (1) confirm the expectation that branes like monopoles don't annihilate fast enough to disappear before freezeout; (2) to understand the relative annihilation rates of different dimensional Dp branes as a function of p. Such knowledge may be used to motivate preferences for models predicting only a subset of the possible branes (like small dimensional branes, etc.).

Branes and anti-branes which are not parallel have no chance of annihilating and will dominate the energy density of the universe. Thus an upper bound on the annihilation rate can be found by taking all the branes and anti-branes to be parallel, which is what we do in this paper. Although, it would be interesting to, we don't delve into explanations of how the branes parallelize themselves. It may be via some sort of recombination transitions between intersecting branes, or it may occur because the lowest energy configurations allowed by the compact geometry are parallel configurations.

If extra compact directions exist then the density of branes and anti-branes at late times will significantly depend on the process of decompatification of the large dimensions or compactification of the small dimensions, the initial topology of spacetime, and more generally the evolution of the compact directions. Our results can be summarized as follows.

Stabilized compact directions and 4 expanding large directions: because of strong brane anti-brane interactions, branes filling the non-compact directions and separated only in the compact directions will quickly annihilate. Such branes will not feel the cosmic expansion which typically causes interactions to freeze out below some temperature. Branes with $p \geq 3$ which completely fill the non-compact directions and possibly wrap some of the compact directions will not pose any cosmological problems. Interactions between branes with $p < 3$ will be frozen out, and these branes may pose severe cosmological problems. Branes with $p \geq 3$ which do not fill the non-compact directions will pose problems as well.

Expanding compact and non-compact directions: branes of all dimensionalities will soon dominate the energy density of the universe and cause it to collapse. This is because brane interactions will quickly be frozen out and they will be unable to annihilate. If none of the dimensions have been decompactified, such that all the directions are compact but expanding as in the Brandenberger-Vafa model, then a variety of possibilites are possible. If the wrapped branes can generate a positive pressure and confine the compact directions via some sort of dilaton dynamics [3], then all of the wrapped branes may annihilate. This will ameliorate any brane problem but also prevent wrapped branes from allowing four compact directions to grow large. If the compact directions are not stabilized, then interactions will be frozen out and branes will be littered throughout spacetime.

do not affect the gravitational dynamics of the universe in this context. This can be seen in a different manner. The perturbation δg_{00} of a massive object on the surrounding spacetime in linearized gravity is given by the newtonian potential,

$$\delta g_{00} = -\frac{G_d m_{Dp}}{r^{d-2}} \longrightarrow 0 \qquad \text{if } g_s \to 0 \tag{1.1}$$

because the $d+1$-dimensional Newton's constant varies as $G_{d+1} \sim g_s^2$ and $m_{Dp} \sim 1/g_s$.

One interesting, but expected result from our calculations is that high dimensional branes annihilate more efficiently and drop out of equilibrium later because they interact more vigorously with other branes and matter in the bulk. Thus a type-IIB universe initally filled with branes may migrate to a universe filled with only a few $D3$ branes and many $D1$ strings. This may be very attractive from a braneworld point of view.

The plan of this paper is as follows. First we consider the idealized case of non-interacting branes in a static universe; this is essentially a reworking of the heuristic arguments of [4] from a somewhat more algebraic point of view. Then we consider the case of interacting branes in a static universe. Next, we move onto the more realistic case of interacting branes in an expanding universe. Finally, we end with a catalog of how our results apply to scenarios with differing numbers of compact directions, expanding directions, wrapped directions, etc..

1.1 Non-interacting branes in flat space

The point of this section is to place on a more rigorous footing, the well known result of [4], that two noninteracting objects each of spacetime dimension $p+1$ will meet in D spacetime dimensions only if $2(p+1) \geq D$.

If we ignore long-range interactions and the cosmic expansion rate H^{-1}, then the annihilation rate of a system of branes in a static spacetime M, should depend only on the distinguishing features of the branes such as their dimensionality $(p+1)$, and the geometry, topology and dimensionality of the spacetime M.

Consider a non-coincident Dp brane and $\bar{D}p'$ anti-brane. Label them as brane A and brane B and their worldvolumes as A and B. A and B will annihilate if they are coincident, parallel, and if $p = p'$. Anihilation occurs because tachyons arise which eventually condense to the minima of their potential. The minima for coincident anti-parallel branes corresponds to the disappearance/confinement of all open string degrees of freedom – i.e. annihilation [5]. Tachyons also arise if two branes intersect at angles [7]. However, the minima of the tachyon potential in that case correspond to changes in the geometry of the branes and not annihilation.

A Dp brane and $\bar{D}p'$ anti-brane will only meet each other if their spacetime paths intersect at some point in time — i.e. if their worldvolumes intersection We have the following set theory identities

$$A + B = A \setminus B + B \setminus A + A \cap B \tag{1.2}$$

where

$$\begin{aligned} A \setminus B &= A - A \cap B \\ B \setminus A &= B - A \cap B\,. \end{aligned} \tag{1.3}$$

This allows us to write

$$\begin{aligned} \dim(A + B) &= \dim(A - A \cap B) + \dim(B - A \cap B) + \dim(A \cap B) \\ &= \dim(A) + \dim(B) - \dim(A \cap B) \end{aligned} \tag{1.4}$$

where $\dim(A \cap B)$ is appropriately defined to ensure that the dim operator is linear.[2] We now use the definitions

$$\dim(A+B) \leq D\,; \qquad \dim(A) = p+1\,; \qquad \dim(B) = p'+1 \tag{1.5}$$

where D is the dimensionality of the spacetime. We then conclude that

$$\dim(A \cap B) \geq (p+1) + (p'+1) - D\,. \tag{1.6}$$

Thus, brane A and anti-brane B will surely meet at some point in time if $(p+p') > D-2$. Equation (1.6) provides a *lower bound* on the dimensionality of the intersection, and the actual dimensionality of the intersection will usually be larger. For example, if $(p+p') = D-2$, then $\dim(A \cap B) \geq 0$. The only way $\dim(A \cap B) = 0$ and $A \cap B = \emptyset$ can occur is if the worldlines of A and B are roughly parallel - an unlikely situation. Otherwise, they will intersect in a non-zero dimensional space. In the special case when $p = p'$, and the Dp and $\bar{D}p$ are parallel, any intersection of A and B will be $p+1$ dimensional. This is crucial, because annihilation requires the brane and anti-brane to be coincident and that $\dim(A \cap B) = p+1$.

In string theory, $D = 10$. Thus, $D3$ branes will (almost surely)[3] not meet. But, $D5$ and $D7$ branes will meet and annihilate. $D3$ branes and lower dimensional branes may meet. But the number of such configurations is a set of measure zero. For example, intuition tells us that two non-interacting low dimension branes can on rare occasions simply "run into each other." This will occur if brane A and brane B live in a smaller space $W \subset M$, which is a proper subset of M. Then effectively, $D = \dim(W)$. For example, if the worldlines of two $D0$ branes lie in a 2D plane in a 10D dimensional space, then $W = \mathbb{R}^2$; $D = 2$; and unless the worldlines are parallel, the branes will meet. The probability of two randomly moving objects lying in a proper subset of $\mathbb{R}^{10}$ is small. This would require that a subset of the transverse coordinates of A in M and B in M coincide exactly. The number of such coordinates is $\dim(M-W)$. Thus, the probability, P of lying in the subset is of order

$$P \sim \left(\frac{1}{\mathrm{vol}(\mathbb{R})}\right)^{\dim(M-W)} \tag{1.7}$$

which is vanishingly small. The actual probability will be larger, since A and B cannot be completely random surfaces. They must lie within their respective lightcones.

However, if the transverse dimensions to W are compact with a compactification radius of l, then

$$P \sim \left(\frac{1}{l}\right)^{\dim(M-W)} \tag{1.8}$$

[2] For example, if $\dim(A) = \dim(B) > \dim(A+B)/2$ and the worldvolumes A and B are non-intersecting and parallel, then (1.4) would imply that $\dim(A \cap B) > 0$ which cannot be true. This is ameliorated by defining the intersection of the worldvolumes of two parallel objects to be intersecting in $\mathbb{R}^n$. This is reasonable because generic such worldvolumes will not be parallel and will intersection Alternatively, one can work in projective space where for example, two parallel lines always intersection

[3] In the measure theoretic sense.

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which may still be small, but does not vanish. Thus, compactification helps branes meet. But eq. (1.6) is still the principal criterion determining when non-interacting Dp and Dp' branes will meet.

However, in reality branes are rarely non-interacting. Only parallel branes of the same dimension with the same orientation feel no attraction or repulsion to each other. An inter-brane potential exists for branes at generic angles, branes of different dimensions, and branes and anti-branes [6, 7, 8].

1.2 Interacting branes in flat space

Recall that the charge of a brane relative to another brane depends on the relative orientation. The extreme case is when a brane is rotated by an angle π relative to another brane. This corresponds to an anti-parallel or anti-brane. Just as particles and anti-particles feel a mutual attractive force, so do branes and anti-branes. Two branes with zero relative rotation angle are parallel branes. Thus, it is not surprising that the intermediate case of a rotation by $0 < \theta < \pi$ interpolates between no force and the large attractive force of brane anti-brane interaction; i.e. branes at angles feel forces. However, at a few special angles the force vanishes and the configuration is stable.

We will focus on the force between parallel branes and anti-branes of the same dimension. Only such systems can decrease the brane/anti-brane number densities in the universe.

The potential $\Phi(r)$ between a Dp brane at a distance r away from a $\bar{D}p$ is the analog of the gravitational potential, $\phi(r)$ in four dimensions

$$\phi(r) = -\frac{G_4 m}{r} \qquad \rightarrow \qquad \Phi(r) = \text{constant}\frac{G_{10}\tau_p}{r^{7-p}} \tag{1.9}$$

where G_D is Newton's constant in D spacetime dimensions. In place of the mass, the potential contains the tension τ_p. The quantity $G_D\tau_p$ is analogous to the effective gravity of cosmic strings, $G\mu$, where μ is the string tension. It measures the gravitational strength of the branes. The potential energy $V(r)$, of a brane and anti-brane is therefore $V(r) = m_{Dp}\Phi(r)$ in analogy with the purely gravitational case. Since $\kappa^2 = 8\pi G_{10}$, we can write $V(r)$ as [9]

$$V(r) = -\beta\frac{\kappa^2\tau_p m_{Dp}}{r^{7-p}} \equiv -\frac{h^2}{r^{7-p}}\,. \tag{1.10}$$

Here, β is a numerical factor characteristic of D-branes [9, 10].

$$\beta = \pi^{-(9-p)/2}\Gamma\left(\frac{7-p}{2}\right). \tag{1.11}$$

If some directions are compact then the brane inter-brane potential will depend on the average distance between the branes and anti-branes. If the distance is comparable to the size of the compact directions then the branes will notice the compactness of the extra-dimensional space. Hence, to calculate the effective brane anti-brane potential, the effects of an infinity of image charges will have to be added. This will soften the potential. However, the inter-brane separation is unlikely to be close to the compactification scale

r_c, as a gas of branes with a non-neglible density n, will have an inter-brane seperation of $n^{-(d-p)}$ which will usually be much smaller than r_c — and this is the case that we will investigate.

Because of long-range interactions the naive dimension-based arguments in the previous section do not hold for more general brane systems. Long rang forces lead to infinite scattering cross-sections. Thus branes and anti-branes will disappear due to annihilation if the universe remains static for a sufficiently long time and if annihilation doesn't require impossible topology change of any wrapped D-branes (impossible winding number transitions). Some additional issues related to the interbrane potential are mentioned in appendix A.

1.3 Branes in an expanding universe

We can calculate the brane and anti-brane abundances in an expanding universe as in [13, 14]. Assume that the brane density equals the anti-brane density to ensure charge neutrality. The time evolution of the branes/anti-branes is then given by

$$\frac{dn}{dt} = -nH(d_H - p_H) - \Gamma n^2 \,. \tag{1.12}$$

Here Γ characterizes the annihilation process; dn/dt is the time derivative of the density of branes n; and d_H is the number of expanding dimensions. The constant p_H is the number of expanding directions which the branes fill. If p_s is the number of static dimensions which the branes fill (non-compact or compact), then

$$p = p_H + p_s \tag{1.13}$$

The quantity $(d_H - p_H)$ represents the number of expanding dimensions in which the branes move. The first term in (1.12) represents the dilution effect of cosmic expansion. The second represents brane anti-brane annihilation.

The d_H expanding dimensions may be compact or non-compact. Suppose that they are all non-compact and that the remaining $d - d_H$ spatial dimensions are compact. Then branes living only in the compact directions will still be diluted because their inter-brane separations in the non-compact directions will grow. The only way branes in the compact directions will not feel the first term in (1.12) is if they are all coincident in the non-compact directions which is very unlikely, or they competely fill the extra-dimensions such that $d_H = p_H$. For example, if $d_H = 3$, then a gas of $D3$ branes all lying in the four non-compact directions and separated only in the compact directions will not suffer cosmic dilution. If the extra dimensions are themselves expanding or contracting, then branes will obviously feel additional dilution/contraction.

Interestingly, according to (1.12) branes which fill some expanding directions such that $p_H \neq 0$, will feel less dilution than completely wrapped branes ($p_H = 0$). Thus interactions between completely wrapped branes might be expected to freeze out more quickly than between incompletely wrapped branes ($p_H \neq 0$).

Since branes and anti-branes will annihilate only if they are parallel and share the same transverse space, we will assume that all the branes are parallel and possess the same transverse space $V_\perp$. We can then take the dimension of n to be $[n] \sim [L]^{-d_\perp}$.

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We parameterize the annihilation rate Γ as

$$\Gamma = \frac{A_p}{m^{d_\perp - 2}} \left(\frac{m}{T}\right)^w \tag{1.14}$$

where m is some mass scale and A_p is a numerical factor which we take to be a constant. In principle, A_p will be affected by many issues which we here ignore; see appendix A for a discussion of some of these issues.

If some directions are fixed then the number of expanding dimensions satisfies $d_H < d$. If some of the compact directions are expanding then the expansion rate will be affected by the wrapped branes. For example, the negative pressure of a gas of wrapped branes will increase the expansion rate. If the dilaton is allowed to freely evolve then the branes may actually inhibit and halt the expansion [2]. However, this halting will only aid brane anti-brane annihilation if all of the directions are compact, or if the branes are coincident in the non-compact directions, or they fill the non-compact directions. In general, expanding non-compact directions will still dilute brane densities even if the compact directions stop expanding.

Thus, we take our spacetime to be a FRW universe. We fix the dilaton. We ignore the expansionary effects of the brane density as it increases the expansion rate and freezes out brane interactions more quickly, making a FRW universe a conservative choice.[4] Then the Hubble rate, H, of such a radiation dominated universe is:

$$H = -\frac{\dot{T}}{T} = \frac{T^{(d_H+1)/2}}{C m_p^{(d_H-1)/2}}\,. \tag{1.15}$$

where C^2 is proportional to the effective number of spin degrees of freedom. Integrating (1.12), we find [13]

$$\begin{aligned}\frac{1}{f(T)} = \frac{1}{f(T_i)} + \frac{AC}{w + p_H + (1-d_H)/2} \left(\frac{m_p}{m}\right)^{(d_H-1)/2} \times \\ \times \left(\left(\frac{m}{T}\right)^{w+p_H+(1-d_H)/2} - \left(\frac{m}{T_i}\right)^{w+p_H+(1-d_H)/2} \right)\end{aligned} \tag{1.16}$$

where $f(T) \equiv n/T^{d_H - p_H}$ and is proportional to the comoving density. Here T_i is the initial temperature at which annihilation begins and $f(T_i)$ is proportional to the initial comoving density. If $w + p_H + (1-d_H)/2 < 0$ then the annihilation is cut off for $T \ll T_i$. If instead $w + p_H + (1-d_H)/2 > 0$ as in (1.14) then for $T \ll T_i$ the number of branes $f(T)$ becomes independent of the initial temperature T_i and approaches the limit

$$f(T \ll T_i) \approx \frac{w + p_H + (1-d_H)/2}{AC} \left(\frac{m}{m_p}\right)^{\frac{d_H-1}{2}} \left(\frac{T}{m}\right)^{w+p_H+(1-d_H)/2}\,. \tag{1.17}$$

[4]For higher dimensional branes $\rho = -(p/d)P_{ress}$.

The annihilation rate will depend on brane anti-brane forces and brane interactions with massless bulk fields like the graviton, dilaton, etc. If the bulk is thermalized, numerous closed string fields will populate it. As the branes move toward each other, bulk particles will scatter off of them slowing them down. To scatter a brane by a large angle will require many bulk-particle scattering events, after which the brane's velocity will be randomized apart from a drift velocity in the direction of the attracting anti-brane. Let λ be the distance the brane can travel before it is scattered by a large angle (mean free path). Also let r_c be the distance between a stationary brane and anti-brane for which capture is almost sure. Then, there are two important regimes: (1) the capture distance exceeds the mean free path, $r_c > \lambda$; and (2) $r_c < \lambda$.

In the first case the branes will diffuse toward the anti-branes, and once the branes reach within r_c of the anti-branes they will continue to be scattered such that all velocities transverse to the direct path toward the anti-brane are randomized and effectively zeroed away (impact parameter = 0). The branes will thus collide directly with the anti-branes. If the drift velocity of a brane is sufficiently slow, the brane will be captured by an anti-brane in a bound state and eventually annihilation will occur. Otherwise the brane will simply pass through the anti-brane.[5]

Assuming collision occurs upon capture, the annihilation rate will be the rate of branes diffusing toward the anti-branes. The rate per unit density Γ, will be

$$\Gamma = \frac{F}{n} \tag{1.18}$$

where F is the number of branes approaching the anti-branes per unit time. As in the diffusion of charged particles through a plasma, this rate is

$$F = h^2 \left(\frac{\tau}{m_{Dp}} \right) n \tag{1.19}$$

where τ is the mean free time and is the important quantity determining the rate of brane collisions.

In the second case, once the brane is within r_c of an anti-brane, its transverse velocities will not necessarily be randomized and the brane may streak past the anti-brane. The only way for capture to occur in this regime is for the brane to radiate away its kinetic energy. A moving brane drawn to an anti-brane will radiate via a Larmor-type formula. However, radiative capture is very weak and most branes and anti-branes will survive as we show later.

At high temperature the bulk particles will be very energetic and the mean capture length will exceed the mean free path (case 1). However, as the universe adiabatically expands, the temperature will drop until such a time that brane-bulk interactions are frozen out at a temperature T_f. After $T = T_f$, the mean free path will exceed the capture length and capture will occur only by radiative capture. Hence, below T_f, brane anti-brane annihilation will become very infrequent.

[5]We thank Ian Kogan and others for clarifications regarding solitons passing through one another.

2. Mean free path and freezeout

The capture distance r_c can be obtained by the virial theorem, KE$\sim$-PE. Since KE $\sim mv_{Dp}^2$, and PE$\sim -h^2/r^{7-p}$, we find that

$$r_c = \left(\frac{h^2}{T}\right)^{1/(7-p)} . \tag{2.1}$$

However, $D7$ branes are not asymptotically flat and possess a logarithmically diverging inter-brane potential: PE$\sim -\ln r$. Similarly, $D9$ branes have a quadratically diverging potential: PE$\sim -r^2$. Thus, the capture lengths for $D7$ branes and $D9$ branes are infinite. In any case, the notion of capture length for $D9$ branes is not a meaningful since the branes fill the whole space.

We can obtain the mean free path, λ, for a brane to be scattered by a large angle by bulk fields like gravitons, dilaton, etc., by writing $\lambda = v_{Dp}\tau$, where τ is the mean time between collisions in which the brane is scattered by a large angle and v_{Dp} is the brane velocity. If the branes move with thermal velocities then $v_{Dp} = \sqrt{T/m_{Dp}}$ and $\lambda = \sqrt{T/m_{Dp}}\tau$.

Now, in a time τ, the number of particles which are scattered is

$$N_s = v_a \tau \sigma n_a . \tag{2.2}$$

where σ is the cross section for a particle (graviton, dilaton, etc,) to scatter off of a brane and v_a is its average velocity. A particle can scatter two ways. Suppose the brane fills the $X^1, \ldots, X^p$ directions. Because the brane is an extended object, a particle in the transverse directions may directly strike the Dp brane. Since the brane has a width of order $1/m_s$, unless the incident particle wavelength is smaller than $1/m_s$ it will reflect off the brane (the transmission coefficient is zero) [15]. For a wavelength less than $1/m_s$, the particle will pass through the brane. Thus, for temperatures below the string scale the cross section will be a product of the geometrical size in the $X^1, \ldots, X^p$ directions, which is essentially the volume of the brane V_p, and the cross-section to scatter off of it by streaking by it, σ_s. Thus,

$$\sigma = V_p \sigma_s \qquad \text{for } p < D-2 . \tag{2.3}$$

Note, if $p = D-2$ (e.g. a $D8$ brane), $\sigma = V_p$.

Now, we want the time necessary for scattering by large angles. A particle can be scattered by a large angle when the amount of energy ΔE, imparted to it by thermal collisions is of order the mass of the target object, in this case — the brane mass, m_{Dp}. A gas of thermal particles will impart an energy $\Delta E \sim T$ to the target particle. Thus the target particle will be scattered by a large angle after approximately m_{Dp}/T collisions with lighter (massless) particles. Thus setting $N_s = m_{Dp}/T$ [13]

$$\tau = \frac{1}{v_a \sigma n_a} \frac{m_{Dp}}{T} . \tag{2.4}$$

We have $v_a \sim 1$ and $n_a = aT^{D-1}$ since $D-1$ is the number of spatial dimensions in which the light particles move. Thus

$$\lambda = \frac{1}{a\sigma} \frac{m_{Dp}^{1/2}}{T^{D-1/2}} . \tag{2.5}$$

Note that λ grows with m_{Dp} since the heavier the D-brane is, the harder it is to deflect by bulk scattering.

We can now write down an expression for the temperature at which the mean free path equals the capture length, T_f.

Setting r_c in eq. (2.1) equal to λ in eq. (2.5), we find

$$T_f = \left(\frac{1}{h^2}\right)^x \left(\frac{\sqrt{m_{Dp}}}{a\sigma(T_f)}\right)^y \tag{2.6}$$

where

$$x = \frac{1}{(D-1/2)(7-p)-1} \qquad y = \frac{7-p}{(D-1/2)(7-p)-1} \tag{2.7}$$

We parameterize the scattering cross section, σ_s as

$$\sigma_s = \frac{B}{m^{d_\perp - 1}}\left(\frac{m}{T}\right)^q \tag{2.8}$$

where m is some mass scale. As section B shows for closed string scattering off of branes, $q = 2$ and $m = m_s$. That calculation was done in the zero recoil limit. For compact branes, recoil will be more important and q may deviate from its value $q = 2$.

Using (2.8), we find

$$T_f = \left(\frac{1}{h^2}\right)^{x/(1-qy)} \left(\frac{\sqrt{m_{Dp}}m^{d_\perp - q - 1}}{aBV_p}\right)^{y/(1-qy)} \sim \left(\frac{1}{V_p}\right)^{\frac{x+y/2}{1-qy}}. \tag{2.9}$$

The exponent in (2.9) is

$$\frac{x+y/2}{1-qy} = \frac{1}{9.5 - \frac{1}{7-p} - q}\left(\frac{1}{7-p} + \frac{1}{2}\right) \approx \frac{1}{9.5-q}\left(\frac{1}{7-p}+\frac{1}{2}\right). \tag{2.10}$$

The important feature of (2.10) is that the exponent is strictly positive, ($p < 7$). Using $q = 2$ we find

$$T_f \sim \left(\frac{1}{V_p}\right)^{\frac{1}{7.5}\left(\frac{1}{7-p}+\frac{1}{2}\right)} \tag{2.11}$$

To make the dependence of T_f on p more explicit, we set $V_p \sim \ell^p$ and calculate dT_f/dp. Here, ℓ is some length scale, and the binary relation $\sim$ is used because the different directions may have different sizes.

$$\frac{dT_f}{dp} \sim -\frac{\ln \ell}{7.5}\left(\frac{p}{(7-p)^2} + \frac{1}{7-p} + \frac{1}{2}\right)\left(\frac{1}{V_p}\right)^{\frac{1}{7.5}\left(\frac{1}{7-p}+\frac{1}{2}\right)} < 0 \tag{2.12}$$

Thus $dT_f/dp < 0$. This implies that the higher the dimension of a brane (the smaller the co-dimension), the lower its freezeout temperature is, since V_p grows with p and the exponent in (2.11) also grows with p. This is particularly true if the cycles the branes wrap, or non-compact dimensions the branes fill, are much larger than the string scale. Low dimension branes drop out of thermal contact with the bulk fields first. Higher dimensional branes continue to interact with bulk fields at lower temperatures and are thus captured by the same dimensional anti-branes at lower temperatures. Intuitively, this is plausible because the brane anti-brane potential grows with p, and hence the capture distance grows with p. Also, higher dimensional branes can more easily interact with bulk fields because their extended area is exponentially larger. This is essentially a selection rule which states that a universe populated with branes will get rid of higher dimensional branes more easily. We expect this, since higher dimensional branes can meet much more easily in the non-interacting case and presumably also in the interacting case. This implies that co-dimension dominates over mass in the annihilation process. One might have expected larger mass objects to drop out of equilibrium first. However, because of the extended size of the objects, this appears not to happen. For branes wrapping small compact directions with radii $\sim \sqrt{\alpha'}$, then $V_p \sim 1$, weakening the selection rule. But, in most cases the radii will be at least tens of string lengths. This is because brane formation is a stringy process occurring around an energy $\sim\ m_s$ (energy scale of tachyonic phase transition). Because, of the energy in the momentum modes of the strings and branes wrapping the compact dimensions, the cycles will typically be expanding. Thus by the time the branes have formed and start to look for one another, the compact universe will have substantially grown and the radii will be $\gg \sqrt{\alpha'}$. Furthermore, if we assume that annihilation occurs in the loitering phase of dilaton dynamics as in [3], then radii $\gg \sqrt{\alpha'}$. This is because loitering occurs after the compact universe has grown substantially larger than its early string length size.

From (2.11) we can also infer that for fixed p that T_f grows with the number of static dimensions, p_s, which a brane fills. If we take all such static dimensions to be compact, then p_s is the number of dimensions a Dp wraps. If we write $V_p \sim \ell^p R^{p-p_s}$ where $R(t)$ is a cosmological expansion factor, then

$$T_f \sim \left(\frac{R^{p_s}}{R^p \ell^p}\right)^{\frac{1}{7.5}\left(\frac{1}{7-p}+\frac{1}{2}\right)} \qquad \Longrightarrow \qquad \frac{\partial T_f}{\partial p_s}|_p > 0\,. \tag{2.13}$$

This is expected as (1.12) shows that for fixed p that Dp branes are frozen out more quickly as p_H decreases or equivalently as $p_s = p - p_H$ grows.

Using equations (1.18), (1.19), (2.4) we find that

$$\Gamma = \frac{A_p}{m_s^{d_\perp - 1}}\left(\frac{m_s}{T}\right)^{D-2}. \tag{2.14}$$

Since $d_H - p_H \leq D-1$ and $d_H \leq D-1$, we find that the condition for the number of Dp branes to be independent of the initial number: $w + p_H + (1-d_H)/2 > 0$, is always satisfied since $w = D-2$. Because, T_f is essentially the lowest temperature at which our annihilation mechanism works, we find that at late times the number of Dp branes is

$$f_p(T_f) \approx \frac{D-2+p_H+(1-d_H)/2}{A_p C}\left(\frac{m_s}{m_p}\right)^{\frac{d_H-1}{2}}\left(\frac{T_f}{m_s}\right)^{D-2+p_H+(1-d_H)/2}. \tag{2.15}$$

Note that $T_f/m_s < 1$ and the part of the exponent in (2.15) which is $(T_f/m_s)^{p_H}$ decreases as p_H grows.

This means that the more expanding dimensions the branes fill, the smaller their final quantities are. If no compact directions exist or the branes live only in the non-compact

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directions such that $p = p_H$, then at freezeout the number of high dimensional branes is lower than the number of low dimensional branes for equal initial number of branes. If compact directions exist and the branes wrap p_s directions, then since $p_H = p - p_s$, fewer high dimensional branes still remain. Thus, in addition to the fact that T_f is lower for higher dimensional branes, this further confirms that the number of higher dimensional branes is exponentially suppressed compared to the number of lower dimensional branes.

For a fixed p, since $p_H \propto -p_s$ as a Dp brane wraps more directions its chance of being annihilated decreases. This is because according to (1.12) the more compact directions a gas of branes fills (the fewer compact directions it wraps) the more diluted it becomes. In the limit that the branes fill all of the expanding directions such that $p_H = d_H$, (2.15) can be shown to take the form of $f_p(T_f) \sim 1/\Gamma(T_f)t(T_f)$.

Because f_p does not vanish remnants remain. This is troublesome because once T falls below T_f annihilation can occur only by radiative capture, an inefficient process. For example, a brane may be attracted to an anti-brane. However, to form a bound state, the kinetic energy of both branes need to be radiated away. In the case of monopoles, it is known that almost all orbits leading to capture are nearly parabolic (eccentricity = 1) at the point of closest approach. Most of the radiation is given off during closest approach and the energy loss can change an initially hyperbolic orbit with eccentricity slightly larger than unity to a bound state elliptic orbit with eccentricity slighter smaller than unity. Once captured in this bound state, the brane then spirals in towards the anti-brane. At stringy distances, tachyonic open strings appear leading to brane anti-brane annihilation.

Branes and anti-branes possess Ramond-Ramond charges. Thus, it is not surprising that accelerating branes and anti-branes radiate and possess a Larmor type formula [16]. We write the total power radiated by an accelerating brane/anti-brane as

$$P = \frac{1}{6\pi}\gamma_{p,D}\Omega(D)h^2\dot{v}^2 \tag{2.16}$$

$\gamma_{p,D}$ describes how the power radiated per unit solid angle varies with p, and $\Omega(D)$ is the total solid angle in D spacetime dimensions; $\dot{v}$ is the acceleration which can be found from the trajectory.

We will calculate the radiative capture rate for $D6$ branes wrapped on six compact directions and moving in four non-compact directions. Thus, from a four dimensional point of view, the $D6$ branes behave like point particles. In this case, D is effectively equal to four and $p = 6$. The general case of Dp branes moving in $D-1$ dimensions is not much more interesting. This is because lower dimensional branes have an inter-brane potential which falls off faster and can less easily form bound states. Also, higher dimensional branes like $D7$ branes are expected to disappear quickly since the inter-brane potential of $D7-\bar{D}7$ brane anti-brane pairs does not drop off with distance, but grows logarithmically, $V \sim -\ln r$.

Because we are interested in only the gross properties of the motion, we will approximate the trajectory for a given energy E and angular momentum l by the same trajectory as traced out by a particle with the same energy and angular momentum in four dimensions. We also approximate the orbit during closest approach by an ellipse with eccentricity near unity.

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The radiation of kinetic energy leading to capture has been calculated in the case of monopole anti-monopole annihilation before by [14, 17, 18], and the following calculations are virtually identical. The amount of energy radiated in a frequency interval $(\omega, \omega + d\omega)$ is $dE_r = \omega N(\omega) d\omega$. Here $N(\omega)d\omega$ is the number of quanta radiated with frequency near ω. It can be parameterized as

$$N(\omega)d\omega = \frac{8h^6\gamma_{6,4}}{\pi l^2}(K_{1/3}^2(x) + K_{2/3}^2(x))x dx \tag{2.17}$$

where $K_{1/3}, K_{2/3}$ are Bessel functions and l is the orbital angular momentum, and

$$x = \frac{l^3\omega}{3h^4 m} = \frac{v^2 l^3 \omega}{6h^4 E_k}\,. \tag{2.18}$$

The expectation value of the total energy radiated is then

$$\langle E_r \rangle = \int_0^\infty N(\omega)d\omega = \frac{4\gamma_{6,4}\pi h^{10}}{v^2 l^5} E_k \tag{2.19}$$

where E_k is the kinetic energy. All the kinetic energy is radiated away for angular momentum values less than a critical value, l_c

$$l_c = h^2 \left(\frac{4\pi\gamma_{6,4}}{v^2}\right)^{1/5}. \tag{2.20}$$

Capture will occur for $l < l_c$. This gives a classical cross section of

$$\sigma = \pi \left(\frac{l_c}{\mu v}\right)^2. \tag{2.21}$$

The thermally averaged interaction rate cross-section is therefore (using $v \sim \sqrt{T/m_{D6}}$)

$$\Gamma = \langle \sigma |v| \rangle \approx \frac{K}{m_{D6}^2}\left(\frac{m_{D6}}{T}\right)^{\frac{9}{10}} \tag{2.22}$$

where

$$K = (2^4\pi^7\gamma_{6,3})^{1/5} h^4 \propto g_s^2\,. \tag{2.23}$$

Note that the mass term is not the string mass, m_s, but rather the mass of the $D6$ brane, which is not exceedingly large because the $D6$ was taken to be wrapped on a compact surface.

Since in this case $p_H = 0, d_H = 3$, and $w = 9/10$, we have $w + p_H + (1 - d_H)/2 < 0$. We know from the discussion around (1.16), (1.17) that the annihilation is therefore cutoff and that the brane and anti-brane densities do not appreciably change from their values at $T = T_f$. Thus, the number of branes is not driven to zero and brane anti-brane annihilation doesn't resolve the brane problem. For the opposite case where a $D3$ brane fills the expanding four-dimensional spacetime and moves through the static six dimensional compact space, the annihilation rate Γ will be given by an expression similar to (2.22) and the exponent w is expected to be positive. Then $w + p_H + (1 - d_H)/2 = 14/10 + d_H/2$ will be positive definite because $p_H = d_H$. This means that because the $D3$ branes fill the expanding space their annihilation is not cutoff and they continue to annihilate until they completely disappear. This is despite the fact that brane-bulk interactions are frozen out because the temperature decreases as the universe expands.

3. Catalogue of results

Below we list a somewhat pedantic catalogue of results — how our results apply to different spacetimes. The spacetimes differ in the number of compact, static and expanding directions.

1. *All directions non-compact and expanding:* brane interactions are frozen out and branes are leftover. This corresponds to the case where dynamical compactification of the extra dimensions occurs after brane production.

2. *All directions non-compact and static:* interactions do not have to fight cosmic expansion. Thus all of the branes disappear if the directions are static for a sufficiently long period. This may correspond to an initial state where all the dimensions are initially non-compact, static and some directions are later dynamically compactified.

3. *All directions non-compact, d_H^{nc} are expanding, d_s^{nc} are static; branes fill only static directions:* the distance between branes and anti-branes expands. Hence interactions are frozen out and branes do not annihilate fast enough. (This corresponds to the case of $p \leq d_s^{nc}$, or $p \leq 6$ if $d_H^{nc} = 4$).

4. *All directions non-compact, d_H^{nc} are expanding, d_s^{nc} are static; branes fill only the expanding directions:* this corresponds to $p = d_H^{nc}$ or $p = 3$ for $d_H^{nc} = 4$. As the branes are separated only in the non-expanding compact directions, they do not feel the expansion and interact unhindered by the expansion. Thus if non-expanding directions are static for a sufficiently long period, virtually all of the $D(d_H^{nc} - 1)$ branes disappear.

5. *All directions non-compact, d_H^{nc} are expanding, d_s^{nc} are static; branes fill p_H expanding directions and fill p_s static directions:* as long as the branes move in some expanding directions they do not annihilate efficiently. This only happens if $p_H < d_H^{nc}$.

6. *All directions compact and static:* there is no expansion for interactions to fight. All the branes disappear if the compact directions are static for a sufficiently long period.

7. *All directions compact and expanding:* expansion will eventually kill off interactions and brane anti-brane remnants remain. However, if as in [2, 3] the dilaton is allowed to vary, then via some dilaton dynamics the branes may be able to halt the expansion of the compact directions for a period sufficiently long for them to annihilate.

8. *All directions compact, d_H^c are expanding, d_s^c are static; branes wrap only static directions:* the branes effectively behave like point particles moving in a non-compact spacetime. They do not disappear.

9. *All directions compact, d_H^c are expanding, d_s^c are static; branes fill only expanding directions:* this is again the case of expanding branes moving in a static space. The branes disappear.

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10. *All directions compact, d^c_H are expanding, d^c_s are static; branes wrap p_H expanding directions and wrap p_s static directions:* the compact directions which the branes wrap will act to only dimensionally reduce the branes which move in the expanding directions. This then reduces to scenario 7.

11. *There are d^c compact directions, d^{nc} non-compact directions. The compact directions are static, and the non-compact directions are expanding.* Branes with $p < d_{nc}$ feel the expansion and do not disappear. For $p \geq d_{nc}$ then: (a) if the branes completely fill the expanding directions (e.g. non-compact $D3$ branes in a 4+6 split of spacetime) then they disappear; (2) if the branes only partially fill the expanding directions and wrap p_w directions such $p - p_w < d_{nc}$ then the branes feel the expansion and create a remnant problem; (3) in the unlikely case that all of the branes are coincident in the non-compact directions the brane separation is oblivious to the expansion and the branes disappear.

12. *This is the most general case. There are d^c compact directions, d^{nc} non-compact directions. d^c_H compact and d^{nc}_H non-compact directions are expanding. d^c_s compact and d^{nc}_s non-compact directions are static.* Branes move in the expanding non-compact directions if $p < d^{nc}_H$, and thus do not disappear. If $p \geq d^{nc}_H$ then branes still do not dissappear if they do not completely fill the non-compact space, i,e., if $p - p_w < d^{nc}_H$. If at the same time the branes wrap some of the expanding directions, then either the wrapped branes must halt the expansion of the expanding compact directions or the brane remnants will remain. If they do do halt the such expansion then the brane density tends to zero.

13. *The case of some compact contracting directions.* Contracting directions do not shrink distances in the expanding directions. Thus contraction is unlikely to aid brane annihilation. However, this deserves further investigation.

4. Conclusions

D-branes are attractive brane-world candidates. However, a number of fundamental questions must be answered before they become persuasive alternatives to the standard big bang model. Several of these are: why does there appear to be only one braneworld? Why is that braneworld four-dimensional and not say a $D5$ brane? Why are $D3$ branes favored over higher or lower dimensional branes? If braneworlds were littered throughout spacetime, they would typically over-close the universe. Hence, either other braneworlds are very far away, diluted by something like inflation, or forced to be very rare because of some dynamical anomoly/tadpole cancellation mechanism, or their interactions with matter in the bulk is highly suppressed, or there is only one brane-world — ours.

We showed that it is very difficult to dynamically produce an universe which migrates from a universe filled with many branes to a universe filled with only a few branes. Brane anti-brane interactions are cutoff by the expansion rate and annihilation after that point becomes very inefficient. Dynamical brane anti-brane annihilation in a non-compact universe is in fact more unlikely because branes and anti-branes will typically not be parallel. Hence, brane anti-brane collisions will typically lead not to annihilation but a complicated configuration of intersecting branes.[6]

However, in a universe with d^c extra dimensions which have already been stabilised, we showed that branes may disappear if they fill all of expanding directions, $p \geq d_H^{nc}$. For four non-compact directions, this means that Dp branes with $p \geq 3$ may disappear. However, if the branes do not wrap all of the expanding directions, they will remain. For example, even $D7$ branes may remain if they are separated in only one non-compact expanding direction. Universes in which the expanding directions are compact may become brane-less only if the wrapped branes conspire to halt the expansion. Otherwise, they suffer from the same problems that universes with expanding non-compact directions suffer from.

We found one interesting result: higher dimensional branes tend to annihilate fastest, leaving lower dimensional branes behind. This is intriguing because it implies that in a scheme where branes are dynamically generated as in a tachyonic phase transition that the branes which survive and cause cosmological problems are low dimensional branes like $D3$ and $D1$ branes (in type-IIB theory). Higher dimensional branes disappear. This at least begins to answer the question of why the universe may be filled with $D3$ branes as opposed to higher dimensional branes. This means that only one additional mechanism — inflation — may be needed to make the brane-world idea much more persuasive. Such inflation would dilute the $D3$ branes, making an observer on any $D3$ think that there is only one braneworld. Also, it would dilute any line ($D1$ branes) or point ($D0$ branes) defects on the surfaces of the $D3$ branes. Such inflation would also dilute any small dimensional branes in the bulk as well. It would be interesting to see whether this same result holds once we also include interactions between $Dp - Dp'$ branes, where $p \neq p'$. In general terms this problem is very complex as there will not only be forces between branes of different dimensionalities, but branes oriented at angles, and intersecting configurations attracting other intersecting configurations. We did not include these interactions in the paper as they are irrelevent to the central question asked in this paper: do branes and anti-branes annihilate fast enough to make themselves small components of the energy density of the universe?

Acknowledgments

We thank Carsten Van de Bruck, Fernando Quevedo and Ian Kogan for comments. A.C.D thanks PPARC. M.M also thanks the Cambridge Commonwealth Trust, Isaac Newton Trust, DAMTP, Hughes Hall and PPARC for financial support.

A. Issues related to the interbrane potential, allowed branes and the annihilation rate Γ

Interbrane potential: branes of different dimension feel forces because open string excitations connecting the branes are tachyonic. For example, a Dp is attracted to a $D(p+2)$.

[6] On a compact surface, branes tend to align themselves so as to minimize their energies so that non-intersecting branes wrapping the same cycles become parallel. Thus this may not be true.

The force disappears once the Dp brane lies on the worldvolume of the $D(p+2)$ brane. A Dp is repulsed by a $D(p+6)$ brane, and unlike the $Dp - D(p+2)$ system does not flow toward a stable system [8]. $Dp - D(p+4)$ brane configurations are supersymmetric and feel no forces [6, 7].

A brane and anti-brane in relative motion will have velocity and acceleration dependent potentials as well [11, 12]. For example, the largest velocity dependent piece is

$$\Delta_v V(r) = -\frac{v^2}{2}\frac{h^2}{r^{7-p}} + O\left(\frac{v^4}{r^{7-p}}\right). \tag{A.1}$$

The first piece appears because of supersymmetry breaking and the second is present in susy and non-susy cases. Since both terms change the potential energy in (1.10) by no more than a small numerical factor (of the same sign), we ignore them. We also ignore acceleration terms. Note, we have left out the rest mass $2m_{Dp}$ in (1.10).

Allowed branes: the kinds of branes that can fill the spacetime will be determined by the K-theory charges of the spacetime. However, since D-brane charges have yet to be classified for cosmological spaces, we will simply assume that they are the same as in flat space, i.e., for type-IIA/B theory all even/odd dimensional branes are possible, etc. The kind of cycles which branes can wrap and the number of such cycles are given by the Betti numbers of the compact extra dimensions. The Betti numbers largely dictate which brane configurations are possible. For example, a surface with $b_2 = b_1 = 0$ possesses no one-dimensional or two-dimensional cycles. Hence, $D2/D1$ branes will only be able to exist as contractable two-dimensional/one-dimensional loops on the surface.

A more common compactification like $K3 \times T^2$ possesses 2-cycles from the K3 and T^2, two 1-cycles from the T^2, 3-cycles from combining a 1-cycle of the T^2 and a 2-cycle of the K3, and 4-cycles by combining different 2-cycles of the K3 or by combining a 2-cycle of the K3 with the 2-cycle of the torus. Finally, 5-cycles also arise by combining the entire 4-cycle of the K3 with a 1-cycle of the torus, and 6-cycles by combining the entire K3 with the 2-cycle of the T^2. Thus branes can wrap 1,2,3,4,5, or 6 dimensions.

Annihilation rate Γ*:* The functional dependence of A_p will in principle depend on the following. It will depend on the distribution of winding numbers of each cycle for each type of brane/anti-brane. This is because a brane and anti-brane can annihilate only if their winding numbers, $(n_1, \ldots, n_w)$, around w compact cycles are the same. Collisions between branes can lead to inter-commutation of the branes and a change in their winding numbers. For example, a brane wrapping say cycle γ with winding number $w_\gamma = 1$ and a brane wrapping say cycle β with winding number $w_\beta = 1$ can collide and produce a single brane wrapping both cycles γ and β with winding number $(w_\gamma, w_\beta) = (1, 1)$ if the intersection number of γ and β is nonzero. Thus the intersection numbers tell us which winding number transitions are possible and A_p will generally depend on them.

In the K3$\times T^2$ example, the intersection number $^{\#}(A_r \cdot B_s)$ of a r-dimensional cycle A and a s-dimensional cycle B is one for two 1-cycles, $2b_2(K3)$ for a 2-cycle and 1-cycle. Many intersection numbers are non-zero. This means that branes wrapping the various cycles of the compactification can collide to produce branes with very different winding numbers. If some cycles are isolated, then only branes wrapping those cycles can interact with branes wrapping the same isolated cycles, reducing the volume accessible to such branes. This means that branes and anti-branes wrapping identical isolated cycles interact more easily. However, if the number of branes and anti-branes wrapping an isolated cycle are different, then due simply to topology, complete annihilation of the branes and anti-branes can not occur.

In general, collisions between branes wrapping compact cycles and branes lying along non-compact directions will lead to branes wrapping compact cycles and lying along non-compact cycles [10]. Thus, spacetimes with both compact and non-compact directions are unlikely to have branes lying along only the compact or only along the non-compact directions.

A_p will also be affected by the presence of any points/locii at which the compact surfaces degenerate on the base manifold. At such points the volume of the cycles decreases. Such points may thus act as accumulation points for branes and anti-branes wishing to minimize their energy.

For example, K3 possesses 24 singularities where a T^2 fiber degenerates over a base $\mathcal{B}$ of the K3. Thus, the two dimensions of the branes wrapping the T^2 become very small at the singularities. Presumably at such singular points it is much easier for a brane and anti-brane to annihilate, affecting A_p.

B. Closed string scattering off of D-branes

Closed strings fields such as the dilaton, graviton, anti-symmetric tensor field, and massive modes can interact with a brane by exchanging closed strings. One can think of this as spontaneously created open strings on the brane absorbing and then re-emitting closed strings and then subsequently annihilating themselves. This is the t channel point of view, see figure 1. The dual s channel process is: a closed string strikes the brane, creates an open string excitation which then propagates along the brane and is eventually annihilated when it emits another closed string [19, 20].

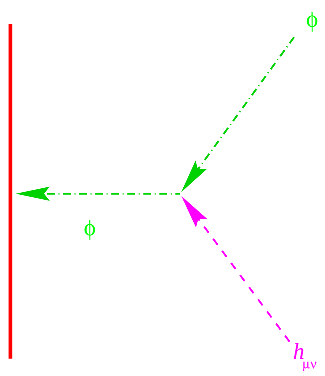

Figure 1: t channel scattering by a brane.

We are concerned most with the t channel because, the t channel scattering amplitude blows up for small scattering angle. This divergence is also common to scattering by other solitons like monopoles.

The form of the scattering amplitude can be guessed as follows. Because of the exchange of a massless particle, there will be a pole and $\mathcal{A} \sim 1/t$ where t is the exchanged momentum squared. The numerator of the amplitude should possess some momentum dependent factors because the interactions between the bulk and the brane occur via quadratic derivative interactions.

Because the amplitude is stringy, there should also be a series of Regge poles, allowing for not only massless closed string exchange, but the exchange of massive closed strings as well. Thus we expect that

$$\mathcal{A} = \frac{a_1}{t} + \frac{a_2}{t+1} + \frac{a_3}{t+2} + \cdots \tag{B.1}$$

where we have taken the poles, in appropriate units, to be $n \in 1, 2, 3 \ldots . = \mathbb{Z}$. The a_i factors are quadratic in the initial and final momenta, p_1, p_2. The amplitude for massless exchange a_1/t, should be proportional the g_s, since for $g_s \to 0$ the brane should decouple from the bulk and no scattering should occur. The simplest way to achieve this is for $a_1 \sim \kappa^2 \tau_p \cdot (\text{function of } p_1, p_2)$, since the amplitude should be proportional to Newton's constant $\kappa^2 \sim g_s^2$.

Because momentum is conserved only parallel to the brane, (in the limit of small recoil), the only invariants are

$$\begin{aligned} s &= 2\alpha'(p_1^2)_{||} = 2\alpha'(p_2^2)_{||} \\ t &= -\alpha'(p_1 + p_2)^2 = -2\alpha' p_1 \cdot p_2 \,. \end{aligned} \tag{B.2}$$

More formally, in the limit of no recoil the tree level stringy amplitude can be found by calculating the disk amplitude with insertions of two closed string vertex operators in the interior of the disk.

$$\mathcal{A} = \int \frac{d^2 z_1 d^2 z_2}{V_{CKG}} \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle \tag{B.3}$$

V_1 and V_2 can be decomposed into two right moving operators and two left moving operators. After fixing the positions of three of them one finds the familiar Veneziano scattering amplitude [19, 20]

$$\mathcal{A} = c_1 \frac{\Gamma(t)\Gamma(s)}{\Gamma(1+s+t)} (sa_1 - ta_2) \delta^{p+1}(p_1 + p_2) \tag{B.4}$$

where $(sa_1 - ta_2)$ is a kinematic factor and is equal to $s\mathrm{Tr}(\epsilon_1 \cdot \epsilon_2)$ if the initial and final state polarisations, ϵ_1 and ϵ_2, are orthogonal to the brane. The delta function insures that momentum is conserved on the brane. When expanded in the t-channel ($t \ll 1, s \gg 1$), the amplitude (B.4) takes the form of (B.1). The constant c_1 is

$$c_1 = \frac{1}{g_o^2 \alpha'} \cdot \left(\frac{\kappa}{\alpha'}\right)^2 \cdot (\alpha'^2) \sim g_s \,. \tag{B.5}$$

The factor on the left is the normalization of the disk amplitude. Note that because the open string coupling constant g_o is basically the Yang-Mills coupling constant, in dimensions other than four it is dimensionful, $1/g_o^2 \sim \alpha'^2 \tau_p$. The second factor in (B.5) comes from the insertion of two closed string vertex operators. The third factor comes from evaluating two sets of propagators like $\langle X(z_1) X(z_2) \rangle = \alpha' \ln |z_1 - z_2|$.[7]

[7] Note the conventions,

$$g_o^2 = \frac{1}{(2\pi\alpha')^2 \tau_p} \,; \qquad \tau_p = \frac{1}{g_s (2\pi)^p \sqrt{\alpha'}^{p+1}} \,; \qquad \kappa^2 = \frac{(2\pi)^7}{2} g_s^2 \alpha'^4 \,. \tag{B.6}$$

The amplitude can also be derived from an effective gravity picture [20]. The NS-NS sector of the low energy action of type-II theories is

$$I_{NS} = \int d^{10}x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{3}{2}H^2 e^{-\sqrt{2}\kappa\phi}\right]. \tag{B.7}$$

The source term for a D-brane is

$$I_s = \int d^{10}x\sqrt{-g}[S_B^{\mu\nu}B_{\mu\nu} + S_\phi\phi + S_h^{\mu\nu}h_{\mu\nu}]. \tag{B.8}$$

To leading order the source functions will be delta function sources at the positions of the branes. For example, the Fourier transform of the dilaton source is $\tilde{S}_\phi(k) = -\tau_p(p-3)/\sqrt{8}$. If one expands the metric as $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$, we can read off the dilaton-dilaton-graviton vertex $\tilde{V}_{h\phi\phi}$ as follows

$$2\kappa h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi \to 2\kappa\epsilon_{\mu\nu}p_1^\mu p_2^\nu\phi(p_1)\phi(p_2) \tag{B.9}$$

and find that the amplitude

$$\mathcal{A} = i\tilde{S}_\phi(t)\tilde{G}_\phi(t)\tilde{V}_{h\phi\phi} \tag{B.10}$$

is the same as the massless part of (B.1) and (B.4) if we perform a field redefinition $h_{\mu\nu} \to 2\kappa h_{\mu\nu}$ and $\phi \to \sqrt{2}\kappa\phi$.

The high energy limit, $s, t \gg 0$ of the amplitude (B.4) is exponentially damped because of the Regge poles ($\mathcal{A} \sim s\exp(-\alpha' s m(\phi)/2)$ [21]. However, it peaks in the small momentum transfer limit $t \ll 1$, but large energy limit $s \gg 1$. In that case

$$\mathcal{A} = c_1\frac{s}{t} = \frac{c_1}{2\sin^2\frac{\theta}{2}} \tag{B.11}$$

where we have assumed that momentum of the incoming particles to the brane such that $s = \alpha' E^2$. This is the massless term in our heuristic derivation (B.1). The above amplitude is the zero recoil limit. In this case, $p_1 = p_2$, because as in usual Compton scattering for $m_{Dp} \to \infty$ [22]

$$\omega_2 = \frac{\omega_1}{1 + \frac{\omega_1}{m_{Dp}}(1-\cos\theta)} \xrightarrow{m\to\infty} \omega_1 . \tag{B.12}$$

Thus $t = p_1 \cdot p_2 = \alpha' E^2(1-\cos\theta)$. Because of this and momentum conservation, the final state $|f\rangle$ will be identical to the initial state $|i\rangle$. The probability of scattering is

$$\mathcal{P}(i \to i) = \frac{|\langle i|S|i\rangle|^2}{\langle i|i\rangle\langle i|i\rangle} = \frac{|\mathcal{A}|^2}{2p_1 \cdot 2p_1} \sim \frac{g_s^2}{E^2\sin^2\theta/2} \tag{B.13}$$

where we have explicitly included the normalization, $\langle i|i\rangle = \langle p_1|p_1\rangle = 2E_{p_1}$.

The cross section will be proportional to $\mathcal{P}(i \to i)$ once we average over the angle θ. Thus

$$\sigma_s \sim \frac{g_s^2}{E^2}\frac{1}{2\pi}\int_{\theta_{\min}}^{2\pi}\frac{d\theta}{\sin^2\theta/2} \sim \frac{g_s^2}{E^2}. \tag{B.14}$$

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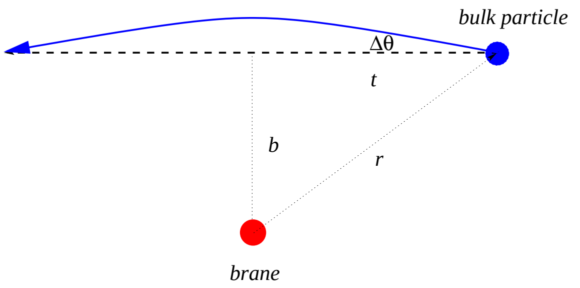

Figure 2: Scattering of a bulk particle by a brane. Note, t is time, and r is the distance from the brane to the bulk particle.

The dimensionful factors ignored in (B.14) are simply factors of α' and are not so important in determining the energy dependence of σ_s. The integral in (B.14) diverges for $\theta_{\rm min} = 0$. Now a small deflection angle corresponds to a large impact parameter [23]. However, because the bulk fields scatter off other bulk fields as well as the brane, the maximum impact parameter of a particle which solely scatters off the brane is bounded. A bulk particle will feel a long range force from other bulk particles. The momentum change due to brane-bulk scattering will be (see figure 2)

$$\begin{aligned}\Delta p &= \int F dt \sim \int \frac{dt}{r^{8-p}} = \int_0^\infty \frac{dt}{(b^2+t^2)^{(8-p)/2}} \\ &= \frac{1}{b^{7-p}} \int_0^\infty \frac{d(t/b)}{((t/b)^2+1)^{(8-p)/2}} \sim \frac{1}{b^{7-p}} \end{aligned} \tag{B.15}$$

where b is the impact parameter and t is time. The angle of deflection will be $\theta = \Delta p/p$. If the mean free path of the bulk fields is ℓ, and $p \sim T$, then $\theta_{\rm min} \sim 1/(T\ell^{7-p})$. Hence, the forward scattering limit of (B.14) is not singular.

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